Small signal modeling of hysteretic current mode control using the PWM switch model

J. H. Park
Seoul National University
Seoul, Korea
E-mail: whoishe@snu.ac.kr

B. H. Cho
Seoul National University
Seoul, Korea
E-mail: bhcho@snu.ac.kr

Abstract - In this paper, based on the small signal analysis of switching power converters under a hysteretic current mode control, a unified PWM switch model is derived. The model is easy to handle because all manipulations are performed on a circuit diagram which includes simply-described function blocks indicating the physically intuitive relationships among the model parameters. This paper presents two kinds of small signal model of hysteretic current mode control such as critical current mode (CRM) variable frequency control and fixed-band current mode control methods. From the results of the PWM switch modeling, the dynamic characteristics of the hysteretic mode control scheme and that of the peak current mode (PCM) control are compared and summarized. Finally, both of the hysteretic current mode control models are verified by the experimental results.

I. INTRODUCTION

Since hysteretic mode control was introduced to the converter controller in 1967 [1], it has been researched as a good alternative for regulating the current or voltage of a switching converter due to its fast dynamic characteristics and easy implementation. Not only the excellent loop performance it has, but also the instability can be prevented for all duty cycle since hysteretic control is basically based on the bounded operation between upper and lower trip point. For more detail, some considerable features of the hysteretic mode control are summarized as follows:

- Hysteretic control has the smallest phase lag of all the current-mode control methods that have been presented, and it is completely free from short-circuit-current runaway [2].
- The intrinsic advantage of hysteretic regulators is the smallest number of parts, resulting in high reliability and low cost [3].
- Practical hysteretic regulators always comprise an additional current-limiting loop for switch protection [3].
- Compared with the hysteretic control cell, not only constant frequency control, but also constant-on (off) time control has the larger phase shift near one-half of the switching frequency because of the sample and hold effect in the current loop.

In spite of those advantages, the theory of operation is rather involved and no straightforward design or analysis method is available, which makes converter designers hesitate in using this method. In this paper, two kinds of hysteretic control methods will be considered: Critical conduction mode (CRM) control and fixed hysteretic band control. Critical conduction mode control is a switching converter control method that operates on the boundary between continuous conduction mode (CCM) and discontinuous conduction mode (DCM). The behavior of the hysteretic control can be explained physically by the fact that both the peak and valley currents are directly commanded [3], and CRM control is the case where the valley current command is fixed at ground. The injected current is controlled within the hysteretic window and independent from the given input / output conditions up to half of the switching frequency. Instead of varying hysteretic band such as CRM control, the fixed hysteretic band control has a variable valley current command for maintaining the fixed hysteretic window range, the regulators of which are variously described as free-running, asynchronous, or simply as bang-bang regulators [4]. From the following section, the small signal model of the two hysteretic model control will be established using a PWM switch cell for dynamic analysis and will be verified by the hardware experiments.

II. SMALL SIGNAL MODELING OF CRITICAL CONDUCTION MODE CONTROL

Most of the conventional small signal modeling focuses on the converter dynamics of the fixed frequency operation. The operation of the switching converter consists of the subintervals within a switching cycle and they mostly can be analyzed by the state space averaging approach based on the operating modes distinguished by the inductor current slope [5]. A small-signal dynamic model is obtained by averaging, perturbation, and linearization of the large signal state-space result within a switching cycle. In the fixed frequency PWM scheme, the state space averaging method is able to produce a duty-to-output transfer function due to the linearity of duty cycle. In variable switching frequency converter modeling, however, there has been some limitations to apply the state-space averaging technique because the duty cycle is nonlinear to the control voltage (command). Recently, since some self-oscillating variable frequency topologies have emerged with very different system dynamics from the fixed frequency one, a new modeling method has been needed in order to describe the...
variable frequency action [6,7]. After this issue emerged, a new modeling approach was introduced by Suntio that extends the state-space averaging method from a fixed frequency to a variable frequency by taking additional conditions that explain the variable frequency effects. The extended method replaces the duty cycle with other parameters, such as $T_2$ and $T_{ON}$, which are linear even in the variable switching frequencies [8]. This extended state-space averaging approach has already been proved by applying it to a few basic converters under hysteretic current CRM control [7]. However, the derivation procedures depending on the analytical calculation require very tedious and complicated steps, and the final equations are not simple and it is difficult to find the physical meaning. On the other hand, the PWM switch model is very easy to handle because all manipulations are performed on the circuit diagram, instead of on its equations. Hence, the circuit averaging technique gives more physical interpretation to the model and allows the small-signal ac model to be written almost by intuition with very simple forms [9]. Another important advantage of the PWM switch modeling is that the same model can be used in many different converter configurations. This generality comes from the physical interpretation and enables the model unification of many derived circuits with ease. From the next sections, Suntio’s method will be briefly introduced and the proposed PWM switch model will be derived as well.

A. **Suntio’s method**

Let us use a hysteretic current CRM converter as an example. To derive a small-signal model with the state-space averaging approach, the time-varying averaged inductor current and the output capacitor voltage are chosen as state variables, the input voltage and the output current as input variables, the output voltage and the input current as output variables, and the peak inductor current as control variable. Figure 1 shows the inductor current waveform of the CRM converter (for buck, boost, and buck-boost). The derivative of the time-varying averaged inductor current waveform in Fig. 1 can be approximated by means of the average inductor current slope:

$\langle i_L \rangle' = \frac{t_{on} - t_{off}}{t_s} - \frac{t_{off}}{t_s} \cdot m_2$ \hspace{2cm} (1)

where $m_1$ and $m_2$ are the up and down slopes of the inductor current (see Fig. 1).

![Fig. 1. Instantaneous inductor current waveform of a converter under CRM control](image)

The output voltage $v_o$ and the averaged input current $\langle i_{in} \rangle$ can also be derived as in Eq. (2) and (3).

$\phi_{in} = v + r \cdot C \cdot v'$ \hspace{2cm} (2)

$\langle i_{in} \rangle = \langle i_L \rangle_{on}$ (for buck, boost, buck-boost), $\langle i_{in} \rangle = \langle i_L \rangle$ (for boost) \hspace{2cm} (3)

For convenience, we simplify Eq. (2) with $v = v_o$. However, the effect of the output capacitor ESR ($R_C$) will be reflected in the Laplace domain. After the simplified state equation is manipulated in time domain, we add ESR zero individually in frequency domain [7]. The non-linear state equations and output equations are represented as follows:

$v = \frac{\langle i_L \rangle}{C} - \frac{I_{in}}{RC} \cdot v$ (for buck), \hspace{2cm} (4)

$v' = \frac{\langle i_L \rangle_{on}}{C} - \frac{I_{in}}{RC} \cdot v$ (for boost, buck-boost) \hspace{2cm} (5)

$v_o = v$ (for buck, boost, and buck-boost) \hspace{2cm} (6)

$\langle i_{in} \rangle = \langle i_L \rangle_{on}$ (for buck, boost, buck-boost), $\langle i_{in} \rangle = \langle i_L \rangle$ (for boost) \hspace{2cm} (7)

Secondly, the on-time and off-time constraints that reflect the behavior of a converter in CRM are necessary to present the state-space equations as a function of the state variables, input variables, and the control variable [10]. The off-time constraint is derived from the off-time inductor current waveform in Fig. 1 such as:

$t_o = t_{on} + t_{off}$ \hspace{2cm} (8)

where $t_{on} = \frac{L_i}{v_o}$ (for buck, boost-boost), $t_{on} = \frac{L_i}{v_o}$ (for boost).

The on-time constraints can be derived from the relationship between peak current and average current such as $i_p = \frac{\langle i_L \rangle}{2} + \frac{\langle i_L \rangle_{on}}{2}$. The on-time is determined as follows [8]:

$i_p = \langle i_L \rangle + \frac{v}{2 \cdot L}$ (for buck, boost-boost), \hspace{2cm} (9)

$i_p = \langle i_L \rangle + \frac{(v_o - v)}{2 \cdot L}$ (for boost). \hspace{2cm} (10)

Finally, small-signal ac models of the converters under the CRM control are obtained through the perturbation and the linearization procedure of (4)-(10). The state and output equations can be presented in the Laplace domain using matrix form. The matrix equations can be easily used to solve the transfer functions describing the converter dynamics.

The general transfer functions such as control-to-output (eq. (11)), input-to-output (audio susceptibility in eq. (12)), and output impedance (eq. (13)) are presented as (11)-(13).

$G_{v_{in}} = \frac{v}{i_p} = \frac{a_v + e_v + e_s \cdot x}{s \cdot (a_1 + a_2) + s \cdot a_2 a_1 + a_3 a_4}$ \hspace{2cm} (11)
In case of the CRM buck converter, the coefficients for the small signal state equations and the output equations are determined as Eq. (14) and (15):

\[
\begin{bmatrix}
  a_1 & a_2 & a_3 & a_4 \\
  b_1 & b_2 & b_3 & b_4 \\
  c_1 & c_2 & e_1 & c_3 \\
  d_1 & d_2 & d_3 & d_4 \\
\end{bmatrix}
\begin{bmatrix}
  \frac{4}{T_s} \\
  0 \\
  0 \\
  \frac{-1}{D \cdot D'}
\end{bmatrix}
\begin{bmatrix}
  0 \\
  0 \\
  0 \\
  0
\end{bmatrix}
\begin{bmatrix}
  \frac{1}{C} \\
  0 \\
  0 \\
  0
\end{bmatrix}
\begin{bmatrix}
  \frac{-1}{RC} \\
   \\
  0 \\
  0
\end{bmatrix}
\]

(14)

\[
\begin{bmatrix}
  c_1 & c_2 \\
  P_1 & P_2 \\
\end{bmatrix}
\begin{bmatrix}
  \frac{2}{T_s} \\
  0
\end{bmatrix}
\begin{bmatrix}
  0 \\
  0 \\
  0 \\
  0
\end{bmatrix}
\]

(15)

In this section, the derivation procedure of the PWM switch model for the CRM control is presented using the results of Suntio’s method in section II.A. The peak current mode (PCM) CCM control model in Fig. 2 is a very popular example of the PWM switch model. Power stage model is the same as the real circuit network except the nonlinear PWM switching element. The PWM switch model can be used to effectively generalize all the basic topologies and the modulation strategies using small signal block diagram in Fig. 2. Since the PCM control method uses peak current information for the current loop as CRM hysteretic control does, it is anticipated that this model structure can be applied to the hysteretic current mode control method also. From the simplified identification of the invariant structure, the physical interpretations and conceptual extensions can be easily obtained by establishing and comparing hysteretic PWM switch model with PCM one.

\[
G_{v_m,\omega} = \frac{\hat{v}_m}{v_m} = \frac{(a_1 b_2 a_1 + b_2 a_3 s) (1 + sr_c C)}{s^2 - (a_1 + a_4) s - a_2 a_4 + a_4}
\]

(12)

\[
G_{i_m,\omega} = \frac{\hat{i}_m}{i_m} = \frac{(b_1 a_3 - b_1 a_1 + b_3 s) (1 + sr_c C)}{s^2 - (a_1 + a_4) s - a_2 a_4 + a_4}
\]

(13)

B. PWM switch model

Fig. 2 A PWM small signal model for peak current mode control developed by R.Ridley [11]

\[
\hat{i}_m = \hat{i}_m + \frac{2L}{V_c} \left( \frac{\hat{i}_i}{V_c} \right) - \frac{2LI_c}{V_c} \hat{v}_c.
\]

(17)

Substitution of relationship (17) into eq. (16) leads to:

\[
\frac{di_m}{d} = \frac{-2D \cdot L}{V_c} \left( \frac{\hat{i}_i}{V_c} \right) - \frac{2D \cdot LI_c}{V_c} \hat{v}_c.
\]

(18)

In order to remove t on in (18), use (9) to find the relationship as:

\[
\hat{i}_m = \frac{2L}{(V_o - V_c)} \left( \frac{\hat{i}_i}{V_c} \right) - \frac{DT_c}{2L} \cdot \hat{v}_a + \frac{DT_c}{2L} \cdot \hat{v}_b.
\]

(19)

Substitution of relationship (19) into eq. (18) leads to:

\[
\frac{di_m}{d} = \frac{2L}{V_o V_c} \left( \frac{\hat{i}_i}{V_c} \right) - \frac{D}{V_c} \cdot \hat{v}_a + \frac{D}{V_c} \cdot \hat{v}_b.
\]

(20)

Consideration of sensing gain R_s leads to:

\[
\frac{di_m}{d} = \frac{2L}{V_o T_s R_s} \left( \frac{\hat{i}_i}{V_c} \right) - \frac{D}{V_c} \cdot \hat{v}_a + \frac{D}{V_c} \cdot \hat{v}_b.
\]

(21)

From this equation, the parameters of each block are derived as follows:

\[
F_o = \frac{2L}{V_o T_s R_s} = \frac{2DD'}{T_s R_s}, \quad \text{where} \quad k_f = \frac{DT_c R_s}{L} \quad \text{and} \quad F_o = \frac{DD'}{S_k T_s}.
\]

(22)

(23)
\[ k_v \cdot I_o^n = \frac{D}{V_o} \]  \hspace{1cm} (24)

This derived PWM switch model is equivalent to Suntio’s analytical model shown in the previous section. CRM hysteretic control of the buck converter is the same as constant off-time control when only the input voltage is perturbed. Thus, the feed-forward terms of (23) are exactly the same as the terms of constant off-time control.

Compared with the PWM switch model of PCM control, the first feature of the CRM hysteretic model is the absence of sample and hold effect (H \(_e\) block in Fig. 2). Sample and hold effect comes from the sampling action of continuous current feedback which can cause sub-harmonic oscillation especially in \( D > 0.5 \) \cite{12}. This instability is a well-known problem of current mode control and occurs without depending on the converter topology. In order to eliminate this effect, external ramp is added to the current modulator. Parameter \( m_e \), representing external ramp effect, has some relationship with the magnitude of the slope of the sensed current ramp used by the modulator during the on-time of the switch \( S_n \) and the slope of the external ramp added to the modulator \( S_e \), which is \cite{12}:

\[ m_e = \frac{S_e}{S_n} + 1 \]  \hspace{1cm} (25)

This extra work makes the controller design complicated, and the reduced current loop gain gives some loop performance limitations to the controller. In order to design this PCM controller, high frequency model is required even when low-bandwidth system is designed. In CRM case, however, there is no sample-and-hold effect because inductor current reset every switching cycle \cite{13}. Thus, perturbations introduced in the inductor’s natural response are not maintained constant until the next sampling time. Therefore, the controller design requires less time and efforts than for the PCM controller design and its implementation.

Another feature in the model is the peak – average current relation gain block, \( K_v \). In PCM, the gain is unity. On the other hand, CRM has different \( K_v \) \((=2)\) because the perturbation ratio of the peak inductor current value is exactly twice of the average value. This value makes it simpler to derive other control parameters and easier to understand the dynamic characteristics in physical intuition.

Since the structure is very similar to the PCM model, it is easy to compare the two converter dynamic characteristics by obtaining the equivalent condition of the PCM model describing the CRM control action. The PCM control-to-output transfer function is:

\[ \frac{\dot{v}_s}{v_i} = \frac{R}{1 + \frac{RT_v}{L}} \frac{1}{1+\frac{RT_v}{L}(mD'-0.5)} \frac{(1+sCR_v)}{(1+s/\omega_v)}\cdot F_Q \]  \hspace{1cm} (26)

where, \( \omega_v = \frac{1}{RC} \cdot \frac{T_v}{D}\cdot(m, D'-0.5) \), and \( F_Q \) is the term giving the sample and hold effect in the current feedback loop. Since in the buck CRM case, the low-frequency dominant pole is \( \omega_v = \frac{1}{RC} \), the equivalent external ramp effect of PCM is:

\[ m_e, D' = 0.5 \]  \hspace{1cm} (27)

Therefore, the modulation gain is given by

\[ F_o = \frac{1}{m_s T_s} \cdot \frac{2L}{V_o J_v R_v} \]

Based on the results of the previous PWM switch modeling, it is shown that the CRM hysteretic control’s external ramp effect for the buck converter is \( m_e, D' = 0.5 \). Which is equivalent to \( S_e = S_n (D - 0.5) \). At that condition, the change in the inductor current from one sampling instant to the next (Constant frequency trailing-edge PCM control) is:

\[ a = \frac{\dot{i}_s(k+1)}{\dot{i}_s(k)} \frac{S_e - S_n}{S_n + S_e} = 1 \]  \hspace{1cm} (28)

This shows that the current action is on the margin of the stable condition, \( a = \frac{S_e - S_n}{S_n + S_e} \leq 1 \), above which the current perturbation oscillates about the steady-state condition on the alternate switching periods and causes a sub-harmonic oscillation problem \cite{12}. Therefore, the CRM current mode control action can be considered as that of the adaptive external ramp PCM controller changing according on the duty cycle in order to have the greatest current loop gain within every stable operating conditions. The result means that CRM control is one of the most optimally-designed linear current mode control in respect to the trade-off between stability and loop performance. Boost and buck-boost models can be derived by the same procedure as that of the buck model. These modeling results are shown in Table III and they show that PWM switch model is very useful to compare the dynamic characteristics of the two control methods in the physically understandable manner which is very intuitive and efficient to analyze the control action.

III. SMALL SIGNAL MODELING OF THE FIXED HYSTERETIC BAND CURRENT MODE CONTROL

![Fig. 4. Instantaneous inductor current waveform of a converter under the Fixed hysteretic-band (H) current mode control](https://example.com/fig4)

Figure 4 shows the inductor current waveform of a converter under the Fixed hysteretic-band (H) current mode control. H is the difference between the upper trip point (UTP) and lower trip point (LTP) of the Schmitt-trigger comparator. Different from CRM (variable H), this control has a fixed H. This control can also be modeled by PWM switch model as shown in fig. 3 using the same procedure deriving CRM control cases. From the relationship between switching period and on-time as \( \dot{i}_s = \frac{i_s}{\dot{T}_o} \), \( \frac{\dot{i}_s}{i_s} \), \( \frac{\dot{T}_o}{T_o} \), the substitution of the equation
into eq. (16) leads to:

$$
\dot{d} = \frac{2D - L}{T_v} \left[ \ddot{i}_t + \frac{2D}{v_{v}} \dot{i}_t - \dot{v}_{\text{in}} \right]. \tag{29}
$$

In order to remove $t_{\text{on}}$ in (29), find the relationship between the on-time and the state variables, and the result is the same as (19). Substitution of equation (19) into (29) leads to:

$$
\dot{d} = \frac{2L}{V_v T_v} \left[ \ddot{i}_t + \frac{2D}{v_{v}} \dot{i}_t - \dot{v}_{\text{in}} \right] + \frac{D}{v_{v}} \cdot \dot{v}_{\text{in}}. \tag{30}
$$

Considering $R_i$ leads to:

$$
\dot{d} = \frac{2L}{V_v T_v R_i} \left[ \ddot{i}_t + \frac{2D}{v_{v}} \dot{i}_t - \dot{v}_{\text{in}} \right] + \frac{D}{v_{v}} \cdot \dot{v}_{\text{in}}. \tag{31}
$$

From this equation, the parameters of each block are derived as follows:

$$
F_n = \frac{2L}{V_v T_v R_i} \frac{2D}{v_{v}} \quad \text{and} \quad k_i = 1 \tag{32}
$$

$$
k_i \cdot F_n' = \frac{D}{v_{v}} \quad \text{where} \quad k_i = \frac{2D T_v R_i}{L} \quad \text{and} \quad F_n' = \frac{2D T_v}{v_{v}}. \tag{33}
$$

$$
k_i \cdot \dot{F}_n'' = \frac{D}{v_{v}}. \tag{34}
$$

From the results, the control-to-output transfer function is:

$$
\frac{\dot{v}_t}{\dot{v}_v} = \frac{1}{R_i} \left( \frac{1}{s + \frac{1}{2} \frac{T_v}{T_i}} \right). \tag{35}
$$

Therefore, because of the equivalent $m_c$, the fixed $H$ hysteretic control has the same results as the CRM control which is one of the most optimally-designed linear current mode control in respect to the trade-off between stability and loop performance. From these derivation procedures, it is shown that this hysteretic control PWM switch model can be extended into other control method easily, and gives more intuition for the analysis of the complex control action.

IV. EXPERIMENTAL RESULTS

In this section, the experimental results of the CRM current mode control and the fixed $H$ hysteretic current mode control are presented in order to verify the proposed small signal models. Table I shows the major operating parameters of two-converter hardware under CRM control (buck, boost). Figure 5 and 6 show the measurement results of the control-to-output transfer function of the circuits. The measuring equipment was HP4194A network analyzer. The dotted lines in those figures are from the PWM switch model and the solid lines are the hardware results. From the results, it is shown that the proposed model matches the transfer function of the hardware very well. Table II shows the major hardware parameters of the two converters under fixed $H$ control (buck, boost). In the same manner as the CRM cases, figure 7 and 8 show the measurement results of the control-to-output transfer function of the circuits. From the figures, it is shown that the two transfer functions agree well with each other.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Buck</th>
<th>Boost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input voltage ($V_{\text{in}}$)</td>
<td>28V</td>
<td>5V</td>
</tr>
<tr>
<td>Output voltage ($V_v$)</td>
<td>6V</td>
<td>20V</td>
</tr>
<tr>
<td>Peak inductor current ($U_{TP}$)</td>
<td>2A</td>
<td>2.5A</td>
</tr>
<tr>
<td>Switching frequency</td>
<td>35kHz</td>
<td>58kHz</td>
</tr>
<tr>
<td>Inductance ($L$)</td>
<td>120uH</td>
<td>56uH</td>
</tr>
<tr>
<td>Capacitor ($C$)</td>
<td>300uF</td>
<td>230uF</td>
</tr>
<tr>
<td>Capacitor ESR ($r_c$)</td>
<td>35mΩ</td>
<td>40mΩ</td>
</tr>
</tbody>
</table>

Table I. Parameters of the Hardware Prototype of the CRM Control Buck & Boost Converters

Table II. Parameters of the Hardware Prototype of the Fixed H Hysteretic Mode Control of the Buck & Boost Converters
V. CONCLUSION

In this paper, two variable-frequency hysteretic current mode control small signal models are derived with a PWM switch cell. From the PWM switch small-signal model, it is shown that there exist some different features in CRM (and fixed H) hysteretic current mode control from PCM control such as non-subharmonic instability, different feedback and feed-forward parameter gains, so on. Through the mapping of the CRM (and fixed H) control model parameters into the equivalent PCM model with a stabilizing external ramp, it is also shown that both the hysteretic control methods have the optimal dynamic performances in respect to the trade-off between current loop gain and stability. The two kinds of hysteretic control PWM switch model considered in this paper are very similarly described by the PWM switch model, which contributes to the intuitive understanding of the dynamic characteristics of the control methods.