Abstract — In power electronics systems the management of power loss and temperature of switching devices is indispensable for the reliability of the whole system. In this paper, a simple electro-thermal simulation model is presented. This simulation model is capable of predicting the power loss and estimating the junction temperature of power device in various environmental conditions. The electro-thermal model is composed of electrical network model, semiconductor device model and thermal network model. These parts interact with each other to calculate the loss and temperature of device and parameters of each model. By focusing on the slow dynamics of heat sink temperature, the proposed model can be employed for the large time-scale simulations. A 200W boost converter using a power MOSFET as an active switch and adopting a natural convection cooling aluminum heat sink as a cooling device was taken as an example system. The experimental results are compared with the predicted values of the simulation model.

I. INTRODUCTION

Reliability of the power electronics system is mostly determined by the failure of switching devices such as MOSFET, IGBT, and diode. Fundamentally most of these semiconductor devices failure mechanism is thermal run-away occurred when the junction temperature exceeds the maximum allowable temperature. Besides, the operation in higher temperature increases the probability of the device’s failure, although it is below the critical temperature [1]. Temperature rise in the switching device is entirely due to the power dissipation of the device. Also this power loss is affected by the (junction) temperature of the device. Therefore when there needs a power loss management of a device, careful attention must be paid to the thermal information, and the power loss must be taken care of to manage the temperature of the power device.

In device’s microscopic level, various dynamic electro-thermal simulation methods that can handle this interaction between electrical and thermal phenomena have been introduced so far [2]. These can be classified into two groups [3]. The one is called the relaxation method [4] that uses full 3-D FEM (Finite Element Method) coupled with electrical circuit simulator (SPICE, Saber etc). The other is the direct method [5, 6, 7] that computes the thermal information in a coupled manner along with the electrical models. Both of these two methods are adequate for the simulation of short target time order of no more than several seconds. The first one is heavy time consuming task for its large computational complexity. The second one is not a boundary condition free model, so when the time-scale of minutes or hours order is needed it is not applicable because of thermal network’s varying boundary conditions and parameters.

The thermal management device such as heat sink has a very slow temperature dynamics compared to the dynamics of the electrical system. Therefore the power loss of the switching device mounted on the cooling device is controlled by the temperature dynamics of the cooling device. The simulation method adopted in this work focuses on this slow dynamics of thermal management device. By this approach, the proposed model can be employed in the large time-scale electro-thermal simulation. Also it can provide the time varying boundary conditions for the short-time simulation.

II. ELECTRO-THERMAL MODEL

A. Electro-Thermal Model Configuration

Fig.1. Electro-thermal model configuration diagram.

Electro-thermal model accommodates the interaction between the thermal and electrical network. As in Fig. 1, the semiconductor device model is fed with the instantaneous temperature of the device by the thermal network. From this temperature value the semiconductor device model calculates temperature dependent parameters of the device (e.g. on-state resistance $r_{ds, on}$ of power MOSFET). The electrical network uses these parameters to hand over electrical parameters to the
device model. The electrical parameters would be the operating conditions such as voltage drop on the device and current to be drawn from the circuit in which the device is playing a given role. Then in the device model, power loss calculation part gathers all these information to compute the power dissipation in the device. The dissipated power in the device is delivered again to the thermal network for temperature calculation. These steps complete one cycle of the electro-thermal simulation. The emphasis put on in this work is the interval of this simulation cycle.

B. Principal Idea for Large Time-Scale Electro-Thermal Simulation

When calculating the temperature of power device, an equivalent thermal network model like Fig. 2 can be employed. The semiconductor device is usually modeled as a (high order) R-C ladder network for its complicated phase in thermal response. The power loss or heat [W] is represented as current and temperature as voltage in the model. The symbol $R_{th}$ stands for thermal resistance [K/W or °C/W] and $C_{th}$ for thermal capacitance [W·s/K or W·s/°C]. The subscripts $j$, $c$, $hs$, $c-h$ and $amb$ mean junction, case, heat-sink, case-to-heatsink and ambient respectively.

$$P_{loss} = \sum R_{th,i} I$$

$$C_{th,i} \frac{dT_i}{dt} = \sum R_{th,i} I$$

Moreover, the dynamics of the device power loss comes to follow the temperature dynamics of the cooling device. All these facts make it possible to convert a detailed complex model in Fig. 2 (a) to a simplified model in Fig. 2 (b). The junction-to-heatsink thermal resistance $R_{th,j-h}$ in Fig. 2 (b) is the sum of all $R_{th,i}$ ($i=1, ..., n$) and $R_{th,c-h}$. The junction-to-heatsink thermal capacitance $C_{th,j-h}$ is the largest one among the $C_{th,i}$ ($i=1, ..., n$) and $C_{th,c-h}$. This $C_{th,j-h}$ can be neglected if the power loss in the device does not undergo rapid changes in magnitude or in frequency. If does, the effect of $C_{th,j-h}$ should be considered and must be integrated into $R_{th,j-h}$ or $P_{loss}$ in an effective value at a given frequency component. By utilizing this simplified model, the slow dynamics of the thermal network can be focused on and it is enabled to perform a large time-scale electro-thermal simulation.

III. MODEL CONSTRUCTION FOR PRACTICAL SYSTEM

To illustrate the electro-thermal simulation model construction and to perform a simulation, a 200W DC/DC converter with an (natural convection cooling) aluminum heat sink is selected as an example system.

A. Electrical Network

A 200W Boost DC/DC converter is taken for an electrical system and it forms an electrical network. A power MOSFET IRFP450 (TO247 package) is used for a switching device. The converter configuration and package information is depicted in Fig. 3. The electrical parameters of the converter are shown in Table I. The parameters for the IRFP450 MOSFET would be easily obtained from the datasheet.

![Boost DC/DC converter and MOSFET package](image)

Before proceed, an assumption should be set forth. The control loop has been designed properly so that the electrical network responds quickly for the disturbance from the outside...
or for the changes in operating condition compared to the thermal network. In this assumption, it is possible to treat the electrical system as in a steady state operation.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Magnitude</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_g$</td>
<td>28 V</td>
<td>Input voltage</td>
</tr>
<tr>
<td>$V_o$</td>
<td>42 V</td>
<td>Output voltage</td>
</tr>
<tr>
<td>$V_d$</td>
<td>0.8 V</td>
<td>Diode forward voltage drop</td>
</tr>
<tr>
<td>$L$</td>
<td>440 uH</td>
<td>Inductance</td>
</tr>
<tr>
<td>$C$</td>
<td>440 uF</td>
<td>Capacitance</td>
</tr>
<tr>
<td>$r_L$</td>
<td>0.2 Ω</td>
<td>Inductor esr</td>
</tr>
<tr>
<td>$r_C$</td>
<td>0.1 Ω</td>
<td>Capacitor esr</td>
</tr>
<tr>
<td>$R$</td>
<td>8.9 Ω</td>
<td>Load</td>
</tr>
<tr>
<td>$V_{gs}$</td>
<td>14 V</td>
<td>Gate drive voltage</td>
</tr>
<tr>
<td>$R_g$</td>
<td>43 Ω</td>
<td>Gate drive series resistance</td>
</tr>
<tr>
<td>$f_s$</td>
<td>50 kHz</td>
<td>Switching frequency</td>
</tr>
<tr>
<td>$D$</td>
<td>0 ≤ $D$ ≤ 1</td>
<td>Duty cycle</td>
</tr>
</tbody>
</table>

Equation (1), (2) are the steady state (DC) characteristics of the Boost converter.

$$
V_o = \frac{1}{D} \frac{D^2 (V_g - D V_d)}{r_L + D r_{ds,on} + D^2 (r_L || R) + D^2 R^2 / (r_L + R)}
$$

$$
I_L = \frac{V_g - D V_d}{r_L + D r_{ds,on} + D^2 (r_L || R) + D^2 R^2 / (r_L + R)}
$$

If a proper control loop has been closed for the converter, output voltage $V_o$ will be controlled to be a fixed reference voltage by the compensator. Therefore the $r_{ds,on}$ value according to the junction temperature changes duty cycle of the converter and average inductor current $I_L$. This makes the conduction loss to be greatly sensitive to temperature.

### B. Semiconductor Device Model – Power MOSFET

The power loss of a power MOSFET, especially the conduction loss, is very sensitive to the on-state resistance $r_{ds,on}$ of the MOSFET. The $r_{ds,on}$ of a power MOSFET is the most sensitive parameter to the temperature. Consequently, the power loss in the power MOSFET comes to be sensitive to the temperature.

An approximate expression [8] that can be used to predict the variation of the on-state resistance of power MOSFET is

$$
r_{ds,on}(T) = a \frac{r_{300K}}{V_{gs} - V_{th}} \left( \frac{T}{300} \right)^b.
$$

where $T$ is the junction temperature of the power MOSFET in the absolute temperature and $r_{300K}$ is the $r_{ds,on}$ value at $T=300K$ or 25°C. The letter $a$ is the constant whose value is $V_{gs} - V_{th}$. The $V_{gs}$ is the standard (gate-to-source) voltage for which the manufacturer had measured the $r_{ds,on}$. In general, 2.3 is assigned to $a$, but it could vary according to the package type or product. For the selected IRFP450 (TO 247 package), 2.65 is assigned to $b$.

The total (average) power loss of a power MOSFET acting as a switch in power electronics system is the sum of switching loss ($P_{sw}$) and conduction loss ($P_{cond}$). As can be seen in Fig. 4, the conduction loss of a power MOSFET is more sensitive to temperature than the switching loss. The conduction loss has a positive temperature coefficient and rise exponentially as the junction temperature increases. Contrary to the positive temperature coefficient of a conduction loss, the switching loss does not have a fixed tendency to the temperature and changes little in magnitude. Compared with the large variation of the conduction loss, it is no better than a constant.

![Fig. 4. Power loss of a MOSFET versus junction temperature. Total average power loss is a sum of switching loss and conduction loss.](image)

**C. Thermal Network**

The selected IRFP450 power MOSFET has a thermal response (junction-to-case) like Fig. 5. Its step response to the step power input can be obtained in datasheet. The step response can be modeled in the form of (4-1). If the $\gamma=1$, the step response of the thermal impedance can be converted to the rational function of $S$ (Laplace operator) and it is possible to perform a frequency analysis. However, in almost every case of a semiconductor device the $\gamma$ is a non-integer value smaller than 1. The IRFP450 MOSFET has 0.537 for the $\gamma$. The nominal junction-to-case thermal resistance $R_{th,JC}^{nom}$ of the IRFP450 is 0.65 °C/W.

To express the thermal response of a semiconductor device a simple approximation method can be used (4-2). The equation in the form of (4-1) then can be converted to an equivalent R-C ladder network like Fig. 2 (a) or the sum of rational functions

- Details on the power loss calculation could obtained in [10]
of $S$.

$$R_{th,nom}(t) = \sum \frac{\gamma}{\tau} \exp\left(-\frac{t}{\tau}\right)$$

(4-1)

$$R_{th,nom} = \left(1 - \sum \frac{\gamma}{\tau} \exp(-t/\tau)\right)$$

(4-2)

The thermal response to the step power input of the IRFP450 device was estimated using (4-2) – Fig. 5 (a). The $i$ in (4-2) was set to be 7 and estimated with target time range $[10^{-8}, \infty]$ in second. Fig. 5 (b) shows the impulse response of the thermal impedance [°C/W] in frequency domain. This graph resembles that of a low-pass filter. Therefore, it can be predicted that the DC component of a power pulse will pass through without attenuation but the high frequency components will not. The magnitude of the thermal impedance is 0.0036 at 50KHz frequency. That means the 50KHz component of a power pulse that has the magnitude of 277W will results in only 1°C increase.

The selected Boost converter system operates in switching frequency 50KHz. Then the power pulse of a switching device will have the same frequency or higher ones. In conduction loss of the device, a 50KHz component is dominant but its relatively small magnitude order of tens of Watt will make no effective ripple value. In switching loss, its peak instantaneous power is (maybe much) higher than that of a conduction loss. It will undergo however, more strong attenuation than the conduction loss because of its very short time duration or very high frequency components. Consequently, both of these high frequency components of the power loss will make no remarkable contribution to the rise in temperature.

In this example system, an aluminum heat sink in natural convection cooling of rectangular fin type was selected as a cooling device. The structure of the selected heat sink is shown in Fig. 6. It is assumed that the heat sink will be placed vertically in the orientation as shown in Fig. 6. The dimensions and properties of the selected heat sink are given in Table II. To represent the characteristics of the heat sink, a Bar-Cohen-Rohsenow empirical model [9] was adopted. The steady state characteristics of the selected heat sink were drawn by using the Bar-Cohen-Rohsenow model in Fig. 7.

![Fig. 6. Structure of the selected heat sink](image)

Natural convection cooling, rectangular fin type, aluminum heat sink

<table>
<thead>
<tr>
<th>Dimension and properties of the selected heat sink</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>82.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Properties</th>
<th>Density ($\text{Kg/m}^3$)</th>
<th>Specific heat ($\text{J/Kg°C}$)</th>
<th>Conductivity ($\text{W/m°C}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.697×10^{3}</td>
<td>878.4</td>
<td>168.6</td>
</tr>
</tbody>
</table>

In heat transfer theory, the relationship between the heat $Q$ [W] and the temperature $T$ [K or °C] is represented as (5) – Newton’s law of cooling. Where, the symbol $T_{amb}$ is ambient temperature, $A$ is the surface area [m$^2$] of the material and $h$ is heat transfer coefficient [W/m$^2$K or W/m$^2$°C]. Then thermal resistance $R_{th}$ [K/W or °C/W] is defined as (6).

$$Q = hA(T - T_{amb})$$

(5)

$$R_{th} = \frac{1}{hA}$$

(6)

As in Fig. 7, the heat transfer coefficient $h$ is not a constant but a nonlinear function of temperature. So is the $R_{th}$ because it is inversely proportional to the $h$. This non-linearity makes it impossible to model a heat sink as a simple R-C network of constant value. This is why the $R_{th,hs}$ in Fig. 2 is represented as a variable resistance.

For the unsteady state analysis of the thermal network, (7) was set up. The symbol $\rho$ is density [Kg/m$^3$], $c$ is specific heat

$$Q = \rho c \frac{dT}{dt}$$

(7)
[J/KgK or J/Kg°C], V is the volume [m³] of the material, and t is time [s].

\[ Q = hA(T - T_{amb}) = -\rho c V \frac{dT}{dt} = -\rho c V \frac{d(t - T_{amb})}{dt} \]

(7)

\[ \frac{T - T_{amb}}{T_0 - T_{amb}} = e^{-t/\tau}, \]

(8)

\[ \tau = \frac{\rho c V}{hA}, \quad R_{th} = 1/hA, \quad C_{th} = \rho c V. \]

(9)

Then this thermal network can be represented as a equivalent R-C network like the cooling device part in Fig. 2. This type of analysis is called lumped-heat-capacity analysis [10]. This analysis assumes a uniform temperature distribution throughout the solid body and this assumption is equivalent to saying that the surface-convection resistance is large compared with the internal-conduction resistance. Such an analysis may be expected to yield reasonable estimates within the error of 5 percent when the following condition is satisfied.

\[ Biot \ number : \frac{h(V/A)}{k} < 0.1 \]

(10)

Where \( k \) is the thermal conductivity [W/m°C] of the material. In almost all case of metal (aluminum, copper etc) heat sink, the dimensionless Biot number is much smaller than 0.1 in practical temperature range. The selected heat sink also satisfies this condition and the value is lower than \( 1 \times 10^{-4} \) – this holds when the heat sink temperature is lower than 250°C and \( T_{amb} \) is room temperature (about 25°C). Therefore, the lumped-heat-capacity analysis will be used to simulate the dynamic response of the heat sink in this work. The Bar-Cohen-Rohsenow model will be used together to generate the \( h \) and \( R_{th} \) according to instantaneous temperature.

IV. SIMULATION AND EXPERIMENTAL RESULTS

To examine the validity of the proposed electro-thermal simulation method, an experiment was performed for the example system. The ambient temperature was 25°C while the experiment was being performed. The experiment was carried out during 154 minutes. During the experiment, one of the operating conditions had been changed to observe the response of the system. After the system had started at \( t=0 \) and the thermal network of the system entered its steady state, the load \( R \) of the Boost converter was doubled at \( t=t_1 \). The load \( R \) was set to its original value after the thermal network had reached its second steady state at \( t=t_2 \).

The simulation conditions and parameters were set equal to those of the example hardware system. The simulation algorithm and electro-thermal model were implemented and simulated with the MATLAB. The time step of simulation was set to be 2 seconds and the simulation was completed in 12.8 seconds with a Celeron 500MHz desktop computer.

Fig. 8. Simulation and experimental results for temperature and power loss. At \( t=t_1 \) load \( R \) was changed from 8.9Ω to 19.8Ω and returned to 8.9Ω at \( t=t_2 \).
The symbol $T_c$ in Fig. 8 (a) is the MOSFET package case temperature. The package case is not a plastic resin part, but the metal plate of the device. The symbol $T_{j0}$ is the temperature of the point on heat sink surface through which the heat (i.e. power loss of the MOSFET) flows from the MOSFET case to the heat sink. This point is the hottest point of the heat sink surface.

The temperatures of the system were measured and recorded with DAQ (Data AcQuisition) scope at every 2 seconds through K-type thermocouples. The ambient temperature $T_{amb}$ was 25°C. The voltage and current waveforms of the MOSFET were dumped (in the form of numerical values - 50,000 samples each) from oscilloscope to PC at several minute intervals. This process was conducted to compute the power loss and on-state resistance of the MOSFET, inductor current and duty cycle of the converter. The dumped voltage and current values within the period of 0.1ms (five switching periods) were multiplied and averaged to get the average power loss. To get the on-state resistance, the voltage waveform values were divided with the current waveform values during the on-time of the MOSFET. Average inductor current was obtained from the average value of the current waveforms during the switching on-time. The duty of the Boost converter was calculated from inspecting the current waveforms.

![Graph of on-state resistance and duty cycle](image)

Fig. 9. Simulation and experimental results for on-state resistance, average inductor current and duty cycle.

At $t=t_1$ load $R$ was changed from 8.9Ω to 19.8Ω and returned to 8.9Ω at $t=t_2$.

The experimental results are presented graphically in Fig. 8, 9 with those of the simulation. Note that the junction temperatures of experimental data in Fig. 8 (a) are estimation values which has been derived from the equation $T_j=T_c+R_{th,j} \times P_{loss}$. These temperatures could not be measured directly in this experiment.

If some slight discrepancies between the instantaneous measured values and simulation results were put away, the simulation results can be interpreted to correspond well with the experimental results. The successful prediction of the trends of temperature, power loss and other parameters would be also manifested by inspecting the results.

V. CONCLUSIONS

Attaching the thermal management device, i.e. cooling device, to the semiconductor device makes the power loss and the temperature of the device to be controlled by the temperature dynamics of the cooling device. In this work, an electro-thermal model for the power MOSFET with a cooling device has been developed. By putting an emphasis on the slow dynamics of the cooling device, a large time-scale simulation was enabled. A Boost converter system with a natural convection cooling aluminum heat sink was configured, experimented on and simulated as an example system. The proposed model was well matched to the experimental results. The results of the simulation and experiment on that system have confirmed that the power loss and the temperature of a power MOSFET did follow the dynamics of a cooling device.

REFERENCES